INTRODUCTION

Hydroacoustic antennas with acoustic screens have been widely used in the construction of hydroacoustic stations (GAS) for various purposes [1 − 6]. They can be implemented in the form of either systems of shielded transducers [7 − 9] or systems with acoustic baffles as separate elements of system designs [1 − 3, 6]. Today, the trend of multifunctionality of modern GAS is becoming more and more popular, when several modes of operation are implemented at one station. Universality necessitates the control of the parameters of the elements that are part of the GAS − transducers [9 − 11], acoustic screens [9], configurations and sizes of sonar antennas [12]. These parameters include directional properties, resonant frequencies, impedances of electrical excitation and radiation, and so on. Methods of managing these parameters are conventionally divided into passive and active. Passive methods include the use of acoustic screens with different physical properties and with different configurations. But their main disadvantage is the difficulty or inability to quickly control the parameters of the antennas and their elements in the process of GAS.

The peculiarity of piezoceramic hydroacoustic transducers is that they are electromechanical devices [12 − 14]. This allows you to quickly control their parameters from the electrical side [9, 10] during the operation of the GAS. The acoustic baffle can also be made in the form of an electromechanical device and its parameters could be quickly controlled during the operation of the station from the electrical side. This is due to its impedance properties depending on the frequency range used. In the range of resonant frequencies, such a screen has the properties of an acoustically soft body, at a distance from mechanical resonances − the properties of an acoustically rigid body, and in the intermediate frequency ranges − the properties of a body with impedance properties [9]. It can be assumed that by electrically exciting the screen, which is made in the form of an electromechanical device made of piezoceramics, the radiation efficiency of GAS can be increased [9, 10].

The purpose of this work is to obtain analytical equations that will quantify the ability to quickly control the parameters of the sonar transducer, located near the acoustic baffle with electrically controlled parameters.

FORMULATION OF THE PROBLEM

We consider the “end-to-end” problem of sound radiation by a sonar antenna made of sonar transducer 1 and acoustic baffle 2 (Fig. 1).
The transducer is a cylindrical piezoceramic shell with medium radius \( r_1 \) and thickness \( h_1 \) with a circular polarization formed by rigid bonding of \( M_1 \) piezoceramic prisms that electrically connected in parallel. Each of the prisms is excited by an electrical voltage \( \psi_1 = \psi_{o1}e^{-i\omega t} \) with amplitude \( \psi_{o1} \) and frequency \( \omega_1 \). The inner cavity of the transducer is filled with an elastic medium with density \( \rho_1 \) and speed of sound \( c_1 \) or can be evacuated \( (\rho_1c_1) \).

The acoustic baffle is a cylindrical piezoceramic shell with medium radius \( r_2 \) and thickness \( h_2 \) with a circular polarization formed by rigid bonding of \( M_2 \) piezoceramic prisms, which electrically connected in parallel. Prisms are excited by an electric voltage \( \psi_2 = \psi_{o2}e^{-i\omega t} \) where \( \psi_{o2} \) is the amplitude of the excitation and frequency is \( \omega_2 \). The inner cavity of the baffles is filled with an elastic medium with density \( \rho_2 \) and speed of sound \( c_2 \).

The antenna is placed in a medium with speed of sound \( c \) and density \( \rho \). The longitudinal axes of the transducer and the baffle are parallel and spaced apart with \( l \). Since, as a rule, the lengths of the transducer and the screen exceed \( 5 - 7 \) of their diameters, the radiation problem could be considered as flat (Fig. 1).

Fig. 1. Normal cross-section of a sonar antenna

Antenna in question performs the conversion of energy and its formation in the surrounding spaces and characterized therefore by the interconnection of all physical fields involved in the emission of sound, and the very processes of conversion and emission of energy. During the conversion of energy in each of the elements of the antenna there is a connection of electric, mechanical and acoustic fields, and during its formation, due to the repeated exchange of radiated and reflected waves between the transducer and the screen, their acoustic interaction.

In mathematical terms, these physical factors are taken into account by a joint solution of the system of differential equations in the coordinates of Figs. 1:

- Helmholz equation describing the motion of elastic media inside and outside the antenna elements:
  \[
  \Delta \Phi_S + k_S^2 \Phi_S = 0; \quad S = 1, 2; \tag{1}
  \]

- motion equations of thin piezoceramic shells with circumferential polarization in term of displacements:
  \[
  (1 + \beta_S) \frac{\partial^2 u_S}{\partial \phi_S^2} + \frac{\partial w_S}{\partial \phi_S} - \beta_S \frac{\partial^2 w_S}{\partial \phi_S^2} = a_S \frac{\partial^2 \phi_S}{\partial t^2}, \tag{2}
  \]
  \[
  -\frac{\partial u_S}{\partial \phi_S} + \beta_S \left( \frac{\partial^2 u_S}{\partial \phi_S^2} - \frac{\partial^2 \phi_S}{\partial \phi_S^2} \right) - \frac{\partial^2 w_S}{\partial \phi_S^2} + \frac{e_{33S} \epsilon_{33S}}{C_{33S}} E_S + \frac{a_S}{\rho_S} \frac{\partial \phi_S}{\partial t} = a_S \frac{\partial \phi_S}{\partial t} ; \quad S = 1, 2; \tag{3}
  \]

- forced electrostatics equations for piezoceramic:
  \[
  E_S = -\text{grad} \psi_S ; \quad \text{div} \vec{D}_S = 0; \quad S = 1, 2. \tag{4}
  \]

Here \( \Delta \) is the Laplace operator; \( \Phi_S \) – speed potential of the \( S \)-th element of the antenna inside \( \Phi_{S} = \Phi_{S} \) and outside \( \Phi_{S} = \Phi \) of it; \( k_S \) – wave number inside \( k_S = k_{1S} \) and outside the element with the number \( S \); \( u_S \) and \( w_S \) – circumferential and radial components of the vector of displacement of the points of the middle surface of the \( S \)-th piezoceramic shell; \( \beta_S = (k_S^2/12\epsilon_{33S})(1 + \epsilon_{33S}^2/C_{33S}^2\epsilon_{33S}^2) \).
The solution of the “end-to-end” problem can be carried out by the method of related fields in multiconnected domains [2, 9]. Let present the mechanical and acoustic fields of the antenna elements by decomposition into Fourier series by angular and wave functions of a circular cylinder:

\[ u_S = \sum_n u_{st} e^{i\eta_n}; \quad w_S = \sum_m w_{st} e^{i\eta_m}; \quad S=1, 2; \]

\[ \Phi_S = \sum_n A_{st} H_{m}^{(1)}(kr_S) e^{i\eta_n}; \quad S=1, 2; \]

\[ \Phi_{1s} = \sum_n B_{st} J_{m}^{(1)}(kr_{1s}) e^{i\eta_n}; \quad S=1, 2; \]

In these equations, the traditional notations of cylindrical functions are used. To determine the unknown coefficients of decompositions (8) and (9), we use relations (1) – (8), pre-expressing the acoustic field of the antenna in its own coordinates of each of its elements. To do this, we need to use addition theorems for cylindrical wave functions [3]:

\[ H_{m}^{(1)}(kr_S) e^{i\eta_n} = \sum_n J_n(kr_{1s}) H_{m-n}^{(1)}(kr_{S}) e^{i(n+1)\eta_n}, \]

where \( r_{1s} \) and \( \varphi_{st} \) are the coordinates of the origin \( O_S \) of the coordinate system of the S-th element of the antenna in the coordinate system of the q-th element.

Algebraization of functional equations (1) – (6) using relations (9) – (12) and properties of completeness and orthogonality of angular functions on the interval \([0, 2\pi]\), allows to obtain an infinite system of linear algebraic equations for determining unknown coefficients of decomposition \( u_{st}, w_{st}, A_{st}, B_{st} \) and as:

\[ -B_{st} J_{m}^{(1)}(kr_{1s}) + i e w_{st} = 0; \]

\[ i e w_{st} - A_{st} H_{m}^{(1)}(kr_{1s}) + \sum_{n=1}^{\infty} A_{st} H_{m}^{(1)}(kr_{S}) e^{i(n+1)\eta_n} = 0; \]

\[ R_{st} w_{st} + \frac{\alpha_0}{h_S} \left( A_{st} H_{m}^{(1)}(kr_{S}) + \sum_{n=1}^{\infty} A_{st} H_{m}^{(1)}(kr_{S}) e^{i(n+1)\eta_n}\right) - \frac{\alpha_0}{h_S} i e \varphi_{st} B_{st} = 0 ; \]

\[ \frac{\alpha_0}{h_S} i e \varphi_{st} B_{st} = \frac{e_{33S} N_S}{2\pi} \int_{0}^{2\pi} d\varphi_{S} ; \]

where

\[ R_{st} = \frac{n^2 (1 + \beta_3 n^2) - (1 + \beta_3 n^2 - \alpha_0 \gamma_3 \alpha_0)(n^2 + \beta_3 n^2 - \alpha_0 \gamma_3 \alpha_0)}{1 + \beta_3 n^2 - \alpha_0 \gamma_3 \alpha_0} ; \]

the bar means the derivative of the function by argument \( r \).

An infinite system of linear algebraic equations (13) can be solved by reduction or fixed-point iteration methods. To do this, the system must be reduced to a kind of quasi-regular by replacing the unknowns \( A_{st} \) and \( B_{st} \) with a new
unknowns \( \bar{A}_{nS} = A_{nS} H^{(1)}(kr_{\bar{S}}) \) and \( \bar{B}_{nS} = B_{nS} J^{(1)}(kr_{\bar{S}}) \).

The numerical values of the unknown coefficients \( w_{nS}, A_{nS} \) and \( B_{nS} \) of the expansions of the physical fields obtained because of solving an infinite system of linear algebraic equations (13) allow to analyze the properties of the hydroacoustic antennas under consideration using the method of numerical experiment. The mechanical equations (13) allow to analyze the properties of the velocity in the form:

\[
\begin{align*}
\frac{\partial u_s}{\partial t} &= \bar{u}_s = -i\omega w_s, \\
\frac{\partial w_s}{\partial t} &= \omega_s = -i\omega w_{sS},
\end{align*}
\]

where \( u_s = \sum n w_s e^{i\omega t}, \ u_{sS} = \frac{i(\beta_s n^3 + n)}{\alpha_s \gamma s \omega^3 - (1 + \beta_s)n^2} w_{sS}. \)

Two expressions are used to calculate the acoustic field outside the antenna. The expression for calculating the field in the near zone of the antenna in the coordinates of the \( S\)-th element of the antenna has the form:

\[
\Phi_i(r, \varphi) = \sum A_{nS} H^{(1)}(kr) e^{in\varphi} + \sum \sum A_{nS} I_{S} (kr) H^{(1)}(kr) e^{in\varphi}.
\]

The expression for calculating the field in the far zone of the antenna is derived from relations (5) and (8) taking into account the asymptotic representations for the Hankel function at \( kr >> 1 \)

\[
H^{(1)}(kr) \approx e^{i(kr - \frac{m}{2} \varphi)} e^{\frac{2}{kr^2}},
\]

and fairness of the next assumptions \( r_s = r - (S - 1)/\cos \varphi; \ \varphi_s = \varphi; \ r_s = r. \) Thus, we have:

\[
\Phi_D(r, \varphi) = \left( \frac{2}{\pi kr} \right)^{1/2} e^{i(kr - \frac{m}{2} \varphi)} e^{\frac{2}{kr^2}} \sum_{s} A_{nS} e^{i\varphi_s} e^{i\omega t}.
\]

The acoustic fields inside the antenna elements are determined by expression (9).

The electric fields of the transducer and the acoustic baffle of the sonar antenna are determined by the following relations.

The expression for the electric excitation current of the antenna elements per unit of their height is determined from the ratio

\[
I_s = \frac{\partial D^{(1)}_{SS}}{\partial t},
\]

where \( S_{SS} \) – the area of the electrode of the piezoceramic prism of the \( S\)-th element of the antenna \( S = 1, 2 \). Taking into account the above formulas for the components of electrical induction, we have:

\[
I_s = -\varepsilon_0 S_{SS} \left[ \sum_{nS} \sum_{rS} \left( \int_{S} \psi_{nS} M_{rS}^2 e^{in2\varphi} M_s + \int_{S} \omega_{sS} h_{nS} M_s \right) \right] \]

The input electrical resistance \( Z_s \) of the \( S\)-th element of the antenna is initiated by Ohm’s law.

**ANALYSIS OF THE OBTAINED RESULTS**

The obtained analytical equations allow making a more rational choice of the construction of the sonar antenna during its design or modernization in the future. As is known [2], the maximum energy efficiency of a sonar station is limited by the mechanical, electrical and cavitation resistance of their sonar antennas. Thus, the mechanical stability of the active elements of antennas limited by the breaking forces that occur in them in operation process and depend on the maximum values of the vibrational velocities of the elements [13, 15]. This necessitates finding numerical values of the amplitude-frequency dependences of the resonant oscillations of the antenna elements.

The electrical strength of the antenna elements, using the existing schemes of organization of their excitation, does not belong to the critical parameters of the antennas. However, knowledge of electric fields, in particular, the input electrical resistances of antenna elements, is essential to ensure creation electronic generators that excite these elements. Knowledge of the magnitude of electric excitation currents allows making a rational choice of materials of communication lines. Note that until recently, strict methods for calculating the electric fields of GAS did not exist [1, 5, 6]. Finally, the ability to quantify the acoustic fields in the near zone of sonar antennas avoids cavitation limitations of GAS, by rationally choosing acoustic pressure levels that do not exceed the threshold of the cavitation-working environment of the station.

Based on the above, we perform a qualitative analysis of the properties of the sonar antenna under consideration, based on the obtained analytical ratios. Such an analysis, although complicated by the presence in their composition of an infinite system of linear algebraic equations (13), is possible because its matrix elements and right-hand sides have a clear physical meaning. First, we note that the system of equations (13) demonstrates the interaction between electric, mechanical and acoustic fields. In this case, it follows from the third equation of the system that the electric energy pumped with the adopted scheme of parallel inclusion of piezoceramic prisms in the shells of the antenna elements only in the zero mode of oscillations of the shells. The parameters of the mechanical fields of both shells, as follows from all equations, are determined by both internal and external acoustic fields, which in turn depend on the densities and speeds of sound of internal and external elastic media.

A special role in the formation of all three physical fields taken by the multiple exchange of emitted and reflected sound waves, which is described in the second and third equations of system (13) by the double sums of \( q \) and \( m \). Their presence indicates the interaction of waves, first, formed by the \( S\)-th and \( g\)-th elements of the antenna, and, secondly, the \( m\)-th and \( n\)-th orders. The nature and magnitude of their impact on all fields determined by multiplier \( H^{(1)}_{m,n}(kr_{gs}) \), which in turn depends on the wavelength \( kr_{gs} \) between the elements. With increasing this distance, the acoustic interaction of the antenna elements weakens, disappearing completely at \( kr_{gs} \rightarrow \infty \). In this case, the sonar antenna is a set of independent elements and its physical fields correspond to the physical fields of a single element. In this case, only the zero modes of their oscillations...
remain in the mechanical fields of the antenna elements, and the elements themselves behave as single-mode transducers. As follows from the second and third equations of the system (13), the decrease in the wavelength $\lambda_n$ is accompanied by an increase in the multiplier $H_{mn}^{(0)}(kr_S)$ and, as a consequence, the effect of double the sum on the mechanical fields of the antenna elements. This effect manifested in the generation in the mechanical field of modes of oscillations following zero. Thus, the antenna elements under consideration converted into multimode oscillatory structures.

Since the right-hand side in the third equation of system (13) has not changed, the energy that is “pumped” into the antenna elements only at the zero mode of their oscillations now distributed between all modes. This leads to changes in the resonant frequencies of the oscillatory systems of the elements, the amplitudes of their resonant oscillations and the bands of their resonant frequencies. Naturally, the acoustic fields, the electric excitation currents and the input electrical impedances of the antenna elements also change. Quantitatively, these changes can be obtained as a result of numerical experiments.

**SUMMARY**

Combining many modes of operation in one GAS requires a review of approaches to the station design and organization of its work. One of them is the transition to dynamic methods of controlling the parameters of the sonar antenna of these stations. The simplest technically this can be realized using the electrical side of sonar antenna transducers. Certain possibilities in the dynamic control of antenna parameters manifested in the replacement of passive acoustic baffles with active piezoceramic shells.

For hydroacoustic antennas formed from a piezoceramic transducer and a baffle, analytical relations are obtained by the method of connected fields in multiconnected areas, which allow quantify the possibilities of dynamic control of the parameters of the hydroacoustic antenna.

A qualitative analysis of these possibilities based on the analysis of the physical meanings of the components of the obtained analytical relations is carried out.

It is shown that in such a statement there are many physical factors and their mathematical equivalents for controlling the dynamic properties of sonar antennas.

**REFERENCES**

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